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This article explores some of the basic physics of bicycling. It helps answer lots of questions about how much energy it takes to power a bicycle over hills, against the wind, and against rolling resistance.

To answer these questions the article relies on a spreadsheet model based on equations from the classic book *Bicycle Science*, by Whitt and Wilson. You can download a copy of the spreadsheet model from the Rivendell site at ___________. Detailed instructions are included there, so I'll skip them for now.

The model lets you punch in the weight of bike and rider, the slope, the speed, and the headwind, and then it tells you the amount of power in watts you are exerting to ride in those conditions. Using a few calculus tricks the model calculates how much an extra pound of weight will cost you in various speed measures at a given power output, and it gives you some additional speed and energy information too. Please, don't take my word for anything that follows. Download the model and prove it for yourself. It’s fun! The model looks like this:

data	
6.00%	gradient (opp/adj)
170	weight of bike and rider (lb)
10.495	speed (mph)
0	headwind (mph)
3.65	frontal area (sq ft)
0.0023	wind resistance constant
0.0051	rolling resistance constant
power info	
250.00	power required in watts
weight info	
-0.049	speed change in mph due to extra pound
0.47%	percent speed loss due to extra pound
4.34	cost in feet/min per excess pound
1.62	seconds lost per mile per excess pound
speed info	
343.02	seconds to go one mile at this speed
15.39	speed in feet/second
55.31	vertical climb rate (feet/min)
energy info	
2.382	kilowatt hours required per 100 miles

POWER: THE BASICS

But first, some explanation about force, work and power, and how they are measured.

Picture a weight of 550 pounds sitting on the ground. Gravity is imposing a **force** of 550 pounds on that weight. If you lift this weight, you are doing **work**. If you lift it exactly one foot, you have done 550 foot pounds of work. Likewise, if you lift a one-pound weight 550 feet, you will also be doing 550 foot pounds of work. To calculate foot pounds, you just multiply the weight by the vertical distance it is lifted.

If you lift 550 lbs one foot over a certain amount of time, you are exerting **power**. 550 foot pounds per second is better known as one horsepower (and one horsepower is roughly 750 watts). Imagine a bicycle-powered elevator weighting 550 lbs. If you can lift the elevator at a rate of one foot per second, you are exerting one horsepower. Got it? Power is the rate of work--it's about moving an object a given distance against a given force in a given time.

One of the main forces a bike must overcome is gravity. When you are climbing you become a bicycle-powered elevator. Assume a large person and their bike together weigh 220 lbs. If they can climb a steep grade at a rate of 2.5 vertical feet per second, then they are exerting one horsepower to overcome gravity.

Although a horse might be able to put out this much power, most humans can't. Some humans could manage it for a few seconds. Very fit athletes can maintain for an hour or so around one-half horse power, or about 375 watts. That's about how much power a strong racer is exerting during a time trial.

The other main force a bike must overcome is wind resistance. This is trickier to understand than climbing force, because the force of the wind hitting you is proportional to the square of its speed. Even when you're standing still, a 20 mph wind pushes against you four times as hard as a 10 mph wind. In general, the force of an object is proportional to the square of its speed--a car moving twice as fast as another takes about four times as long to stop, for example.

On a bike you are pedaling to create the wind, so the harder you pedal, the higher the wind speed, and the stronger the force you must overcome. If you double your speed, the wind force goes up by a factor or four, so you are pushing twice as fast against a force four times as powerful. The power you are exerting goes up by a factor of eight. This lead to the rule that the power required to overcome wind resistance varies as the cube of the speed. This rule also applies to automobiles, and explains why the old 55 mph speed limit saved gasoline. As we'll see, this cube power requirement is a big influence on how much energy you exert when you ride.

Finally, there is rolling resistance. This is due in part to friction losses in bearings, but it is mostly due to tire deformation against pavement. You have to exert extra power to deflect the tire in as it moves around. Rolling resistance is reduced by lower weight, more supple tires, higher air pressures and smoother tire and road surfaces. The model here assumes that rolling resistance is proportionate to weight. It is a relatively small factor compared to the first two (except for mountain bikes, where it can be a much bigger factor).

Now let’s put this all together. Using the model above, let's explore some topics the come up in riding.

CLIMBING--HOW MUCH DOES WEIGHT MATTER?

In the following examples, let's assume a 170 lb. bike and rider combination. This could be a large female or a small male, and includes the weight of the rider and bicycle, plus the weight of clothing, water in the water bottle, any food, tool kit and even sweat in the rider's clothes.

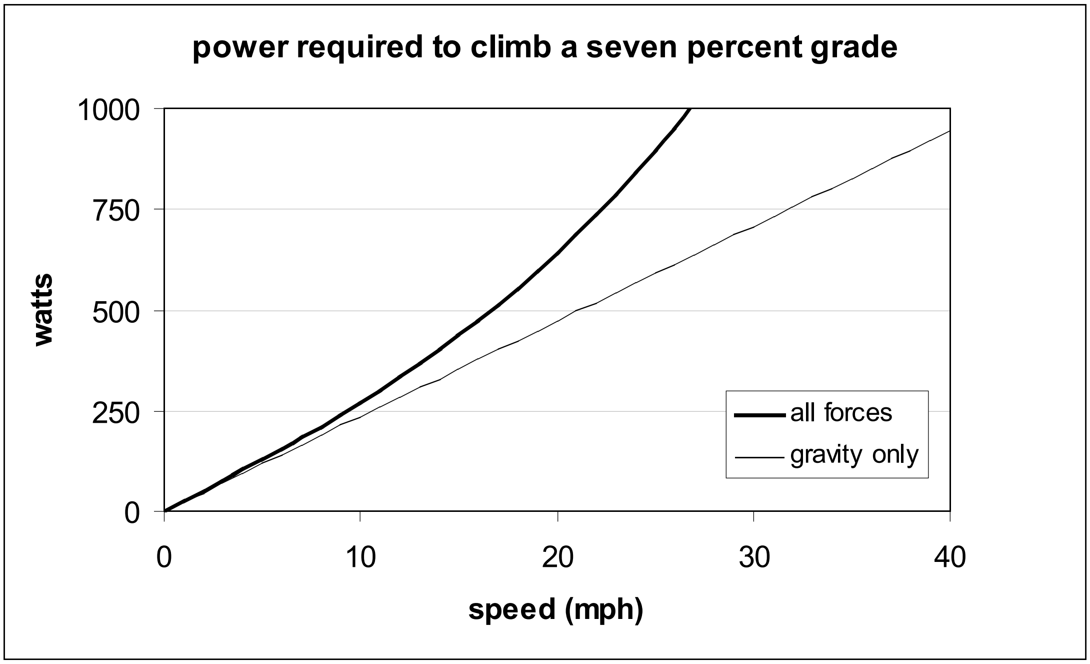
By grade we mean the ratio of vertical height to horizontal distance. A six percent grade means that for every 100 feet you travel horizontally, you climb six feet vertically. A six percent grade makes an angle of about 3.4 degrees from horizontal. By comparison, extremely steep hills--the steepest of the hills in San Francisco, for example--are about 20 percent grades. Those hills angle upward at about 11.3 degrees.

In the example above, our rider climbs a six percent grade with no headwind at a speed of 10.495 mile per hour. This climb requires a power output of 250 watts. The model tells us that one extra pound of weight causes a loss of speed of 0.049 mph, or less than one-half of one percent. Not a huge amount, but it works out to 4.34 feet per minute or 1.62 seconds per mile.

Let's see how this works in a real-world example. Out in Rivendell country, near Walnut Creek, California, the skyline is dominated by Mt. Diablo, with a summit of 3850 feet. The road up Diablo climbs 3260 feet in 11.2 miles. The bottom part of the road is a loop that you can climb from either the north or the south to a ranger station. On a clear Spring day just after a rainstorm this is heaven--the wildflowers are in bloom and you can see all the way to the Sierras. But from there the road to the top can be hellish (they don't call it Diablo for nothing), a short but steep climb of 1670 feet in 4.5 miles, for an average grade of seven percent, including a quarter-mile finish of 17 percent.

Let's say you've made it to the ranger station, and now you are confronting the final climb. It's a warm day and you're pretty sure the drinking fountain at the top is working. You could stash your extra water bottle, your jacket and your food (at about one pound each) behind the nearest bush and save about three pounds. How much difference will three pounds make? If you and your bike together weigh 170 pounds and you push along at 200 watts, in your 36 minute climb to the top of Diablo, you'll save 34 seconds. Is it worth it? I don't know, you decide for yourself.

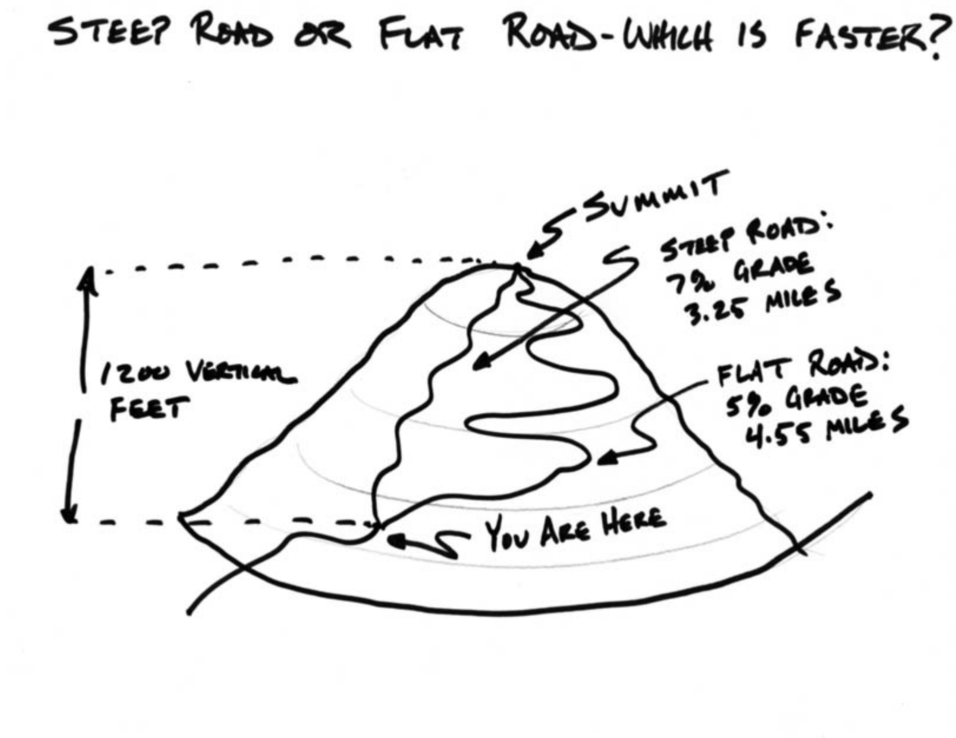
Instead of leaving your gear behind, you could buy a very high-tech, expensive and fragile bike and save weight that way. Or you could just diet and lose three pounds of body fat, which is cheap, since it usually involves eating lower down the food chain and eating out less. For a Tour de France racer, saving 34 seconds in a climb is a big factor. But those riders have almost no body fat to lose, they don't have to pay for their high-tech lightweight bikes, and if the bike should break, there is a car behind them with several spares. For the rest of us, 34 seconds probably doesn't matter that much.



TAKE THE STEEP ROAD OR THE STEEPER ROAD?

When climbing a steep mountain pass, if you want to know how long it will take to get to the top, what's more useful, an altimeter, or an odometer? To answer this question, take a look at the graph above.

The graph shows how much power is required to go a certain speed while ascending a seven percent grade (again, assuming the bike and ride weigh 170 pounds). The straight line denotes how much power you need to overcome gravity alone. The relationship is linear--if you double your power output, you double your speed. When we throw in the other forces a cyclist must overcome, the graph begins to curve upwards. This is due to wind resistance. Once you ride faster than about seven to eight miles per hours, wind



resistance kicks-in, and becomes even more of a factor the harder you ride. But on steep climbs, especially if you are loaded with touring gear, speeds are lower than this, and wind resistance is not significant.

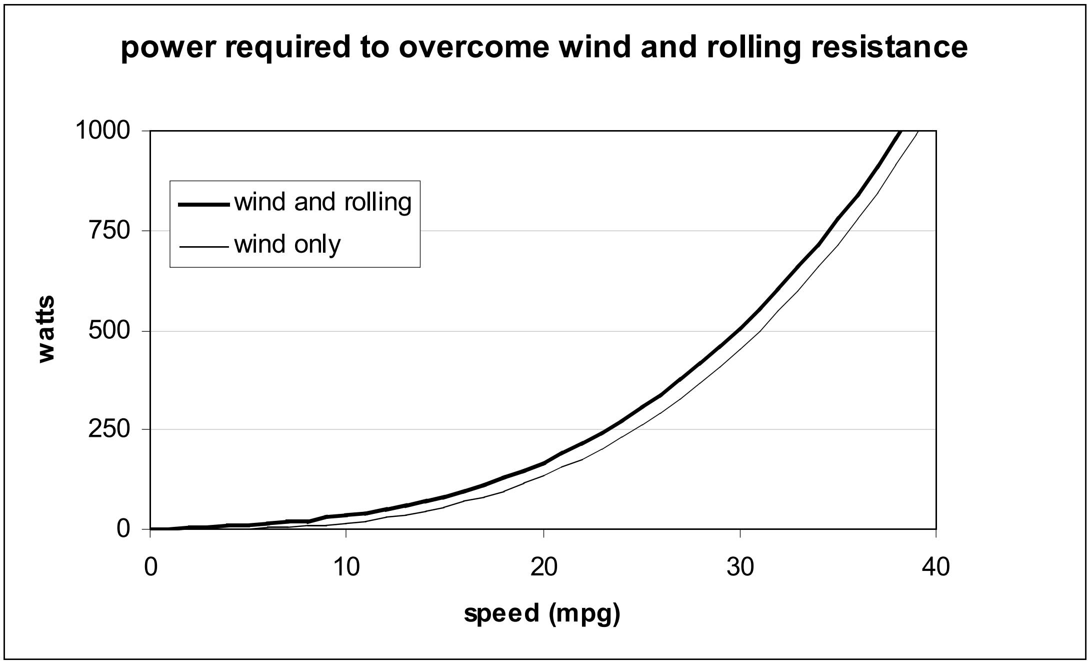
This lack of wind resistance at slow speeds leads to a curious situation in the mountains. Let's do an example. You are touring in the Alps with 30 pounds of extra gear. As you climb a steep mountain pass, you come to a fork in the road. It's late morning, and you are getting hungry, and you are thinking about eating lunch right where you are instead of climbing to the summit first. You need to estimate how long it will take to reach the top. The sign at the fork says the summit is 1200 feet above you (actually the sign would be in meters, but humor me). Your guide book says both roads are equally scenic, but one is a smooth five-percent grade for 4.55 miles, while the other is a steeper seven-percent grade for 3.25 miles. You know your granny gear is capable of handling both grades.

Since this will probably be the only time in your life you ride this road, you want to take it easy and look around of the way up, so you will keep your power output down to 150 watts.

Will one road be much faster than the other? Nope. It really doesn't matter which road you take, because both will get you to the top in 40 minutes, give or take a minute. Because almost all your power output is going to fight gravity alone, if you ride at 150 watts on both roads, your vertical climb rate will be almost identical on both roads--30.5 vertical feet per minute on the seven percent grade, and 29.1 vertical feet per minute on the five percent grade. The extra speed on the flatter road means you are fighting a bit more wind resistance at a speed of 6.63 mph, while on the steep road your speed is 4.97 mph.

On steep slopes and low speeds, you are just a bicycle-powered elevator. Your vertical speed is more important than your horizontal speed. So to answer our earlier question, on the steeps, an altimeter is more useful an odometer. If you know far it is to the summit straight up, and you know what your ascent rate is, you can predict how long it will take to crest the mountain without even knowing how far or just how steep the road is ahead.

WIND RESISTANCE



Now it's time to delve into the cube power rule that governs how hard you must fight wind resistance. The graph above assumes our 170 pound bike and rider, travelling on level ground. There are two curves in the graph above. One shows the power needed to overcome wind resistance only, while the other curve includes rolling resistance, which is relatively minor. That's something to think about next time you feel the urge to buy those skinny tires and pump them up to 120 pounds of pressure. In our model, cutting rolling resistance in half adds less than 1 mph to your speed. Even the best racing tires won't accomplish that much reduction. A speed increase of less than one mph might be the winning margin in a time trial, but for everyday riding it may not be worth the costs of a harsher ride, more flats, and more expensive tires.

The wind-only curve demonstrates the cube power rule exactly. Each time you double your speed, the power required goes up by a factor of eight. As you move from 10 to 20 mph, the power required jumps from 16.7 to 133.5. Instead of thinking about doubling your speed (moving along the horizontal axis) you can think about doubling your power output (moving along the vertical axis). Each time you double your power output, your speed increases by a factor equal to the cube root of two, which is about 1.26. If there were no rolling resistance, a power output of 100 watts would move you along at 18.2 mph. Doubling your output to 200 watts would bump your speed up to 22.9 mph, and another doubling to 400 watts would get you up to 28.8 mph.

These results are modified only slightly by adding rolling resistance. The main point is that the cube power rule of wind resistance means that riding fast sucks up a tremendous amount of energy. This is a fact of life for cyclists, and one the equipment manufactures don't really want you to understand. Even the fanciest carbon fiber aero rims are subject to the very same rules. They let you start with reduced wind resistance but ultimately you are subject to the same ever-steeper wind resistance curve.

The most effective way the thwart the cube power requirement is just to ride slower. Riding slower has lots of other advantages--you can look around and see more, too. Riding slow saves your energy for other things, or let's you tackle longer rides early in the season without working yourself too hard.

ENERGY REQUIREMENTS TO RIDE A CENTURY

One issue with how we've examined wind resistance is that we leave out one fact very much in favor of going faster--you get there sooner. Instead of just calculating watts, let's calculate kilowatt hours, or the total energy required to cover a certain number of miles. Like your electric meter, we can measure energy consumed by multiplying power by the amount of time power is applied. A 100-watt light bulb burning for 10 hours uses one kilowatt hour. A 1000-watt blow dryer left on for one hour also burns one kilowatt hour. Likewise, the spreadsheet model calculates the kilowatt hours required to ride a century.

Now instead of thinking about how many watts it takes to go a certain speed, let's look at the kilowatt hours required to do a certain distance. As we discussed before, doubling your speed means you push four times as hard against the wind at twice the speed, so eight times as much power is required. But you'll get there twice as fast. This leads to the result that on a level ground with negligible rolling resistance, doubling your speed over a fixed distance increased the total energy used by a factor of four. Put another way, kilowatt hours burned over a fixed distance vary as the square of speed. So even over a fixed distance, going faster has a big energy penalty.

TRY A FEW EXAMPLES FOR YOURSELF

Example one:

The cube power rule means steady progress is more efficient that constant changes in speed. The wind resistance power curve is always steeper going faster, and flatter going slower. This means that if you vary your speed during a ride, there is always a penalty compared to going steadily at the average speed. The energy saved when you slow down is never enough to compensate for the extra energy when you go fast. You can use the model to show the same is true for hilly or windy loops. Flat and windless loops are always the fastest.

Example two:

This one stumps many people. Assume you are on level ground, and that rolling resistance is insignificant. You are riding at 10 mph in calm air when a tail wind of 10 mph starts pushing you along. You accelerate to 20 mph. At this speed, you are going twice as fast, but the relative wind is the same as before at 10 mph. Are you putting out the same amount of power, or are you using more?